Type-Based Synthesis of Sound and Complete Random Generators

A Presentation for the REPL 2024 REU

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What are random generators?

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test = \s -> length (take5 s) == 5
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Falsifiable, after 0 tests:
""
```

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instance Arbitrary Int where ...
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Example

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f :: Int -> Int
f x = x + 3
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Example

```
f :: SortedList a -> Nat
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Example

```
f :: BST a -> Nat
```

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Where the type of the program is given, and the task is to generate another program that has that type.

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How can synthesis be type-based?

In type-based synthesis, the goal is to generate a program that satisfies a given type specification.

Introducing Synquid

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Refinement Types

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The Language

```
type Nat = { Int | _v >= 0 }

zero :: { Nat | _v == 0 }
one :: { Nat | _v == 1 }

n :: Nat
n = ??
```

```
n = one
```

```
-----------------------------------------
-- Insertion into a binary search tree --
-----------------------------------------

-- Binary search tree:
-- note how the refinements on the Node constructor define
data BST a where
  Empty :: BST a
  Node  :: x: a -> l: BST { a | _v < x } -> r: BST { a | x

-- Size of a BST (termination metric)
termination measure size :: BST a -> { Int | _v >= 0 } wher
  Empty -> 0
  Node x l r -> size l + size r + 1

-- The set of all keys in a BST
measure keys :: BST a -> Set a where
  Empty -> []
  Node x l r -> keys l + keys r + [x]

leq :: x: a -> y: a -> { Bool | _v == (x <= y) }
neq :: x: a -> y: a -> { Bool | _v == (x != y) }

-- Our synthesis goal: a function that inserts a key into a
insert :: x: a -> t: BST a -> { BST a | keys _v == keys t +
insert = ??
```

```
insert = \x . \t .
    match t with
      Empty -> Node x Empty Empty
      Node x7 x8 x9 ->
        if (x <= x7) && (x7 <= x)
          then t
          else
            if x7 <= x
              then Node x7 x8 (insert x x9)
              else Node x7 (insert x x8) x9
```

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$$\text{Pred} = \{\nu : \text{Int} \mid \nu \geq 0 \wedge \nu \leq 255\}$$

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$$\forall\ \nu\ .\ \nu \geq 0 \wedge \nu \leq 255$$

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Generators (Sound and Complete)

$$\forall\ \nu\ .\ \nu \geq 0 \land \nu \leq 255$$

Synquid (Sound)

$$\exists\ \nu\ .\ \nu \geq 0 \land \nu \leq 255$$

Solution

```
byte :: x: { Int | _v >= 0 && _v <= 255 } -> Int
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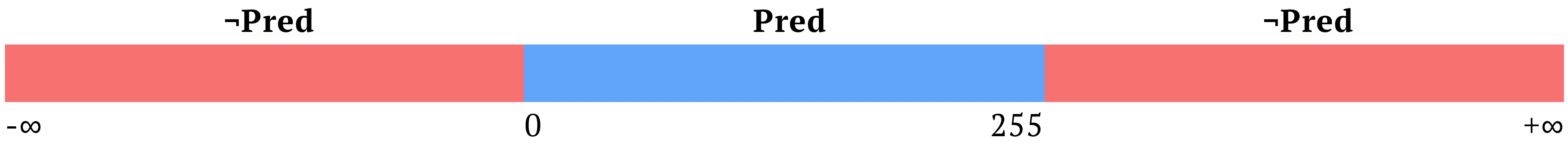

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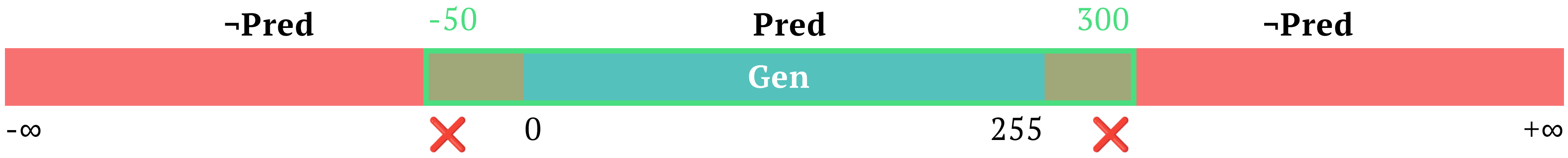

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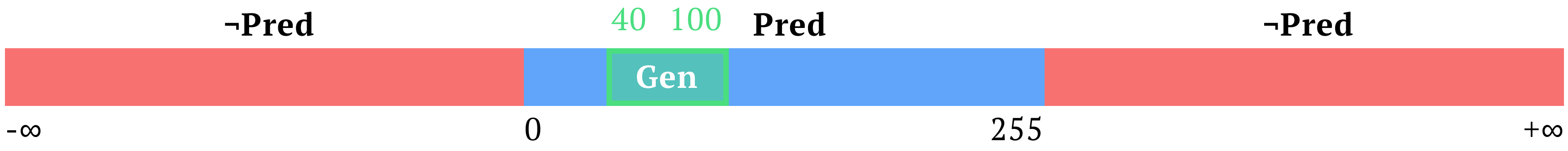

Solution

```
byte :: x: { Int | _v >= 0 && _v <= 255 } -> Int
<byte.x> :: <x.not>: { Int | _v < 0 || _v > 255 }
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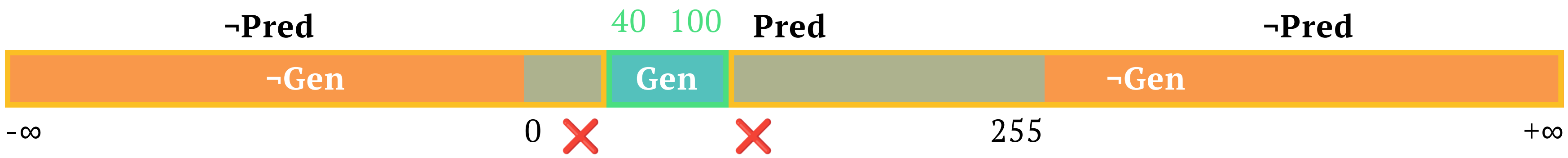
$$\text{Pred} = \{\nu : \text{Int} \mid \nu \geq 0 \land \nu \leq 255\}$$

$$\neg\text{Pred} = \{\nu : \text{Int} \mid \nu < 0 \lor \nu > 255\}$$



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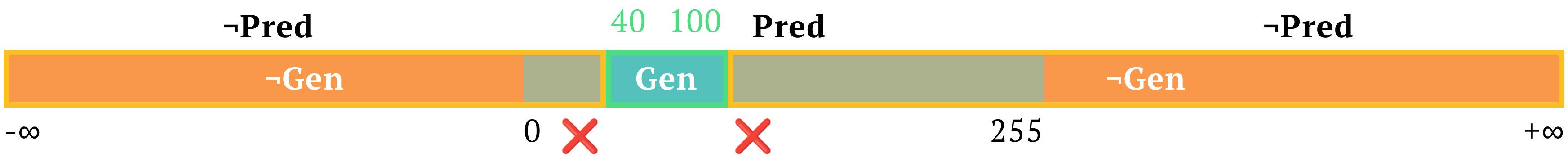

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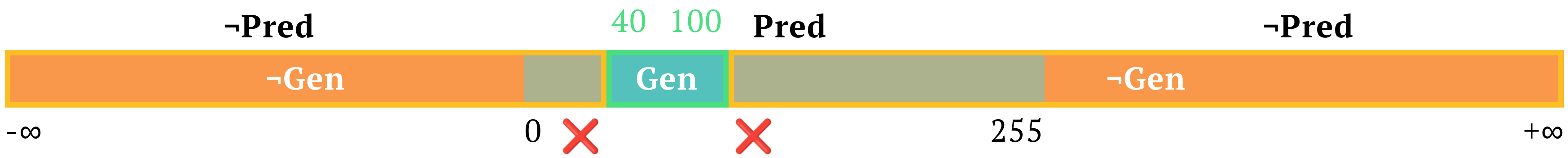

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```
pair :: x: Int -> y: { Int | _v == x + 3 } -> Int
pair_y_not :: x: Int -> y: { Int | _v == x + 3 } -> y_not:

gen_pair :: Int
gen_pair =
  let x = ?? in
  let x_value = g x in

  let y = ?? in
  let y_value = g y in
  let _ = pair_y_not x_value y_value (g_not y) in

  pair x_value y_value
```

```
gen_pair = let x = Eq int in
           let x_value = g x in
           let y = Eq (plus x_value three)
             in
           let y_value = g y in
           let _ = pair_y_not x_value
                     y_value (g_not y) in
           pair x_value y_value
```

- We were able to make Synquid synthesize a sound and complete generators given a refinement type:
  - `byte :: x: { Int | (_v >= 0 && _v <= 255) } -> Int`
  - `pair :: x: { Int | _v <= 3 } -> y: { Int | _v == x + 3 } -> Int`
  - `range :: x: {Int | _v >= -10 && !(_v < -10) } -> y: { Int | _v >= x } -> z: { Int | _v >= y && _v <= 10 } -> Int`

fin.